10d.

THE
New ART

Mathemat. vol 6.

OF
Gauging and Meaſuring
WITHOUT
Inches or Diviſion.

Being Twenty Times more Expeditious, and Ten Times leſs Troubleſome, than the old Way by Inches; each Propoſition proved by the undeniable Principles of Arithmetick, and the Operations wrought both Ways, and Compared for Proof. The whole being Plain and Eaſie, and nothing of the like Nature ever done before, humbly ſubmitted to the Cenſure of all Ingenious Gaugers and Meaſurers.

By *James Lightbody*, P. M.

K

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19. Februar

TO THE
Honourable Commiſſioners
OF HER
MAJESTY's
Revenue of Exciſe,
THIS SMALL
TREATISE
Is humbly Preſented and Submitted to
their Judicious Candor and Protection

By their moſt Devoted,

Humble Servant,

James Lightbody.

THE

PREFACE.

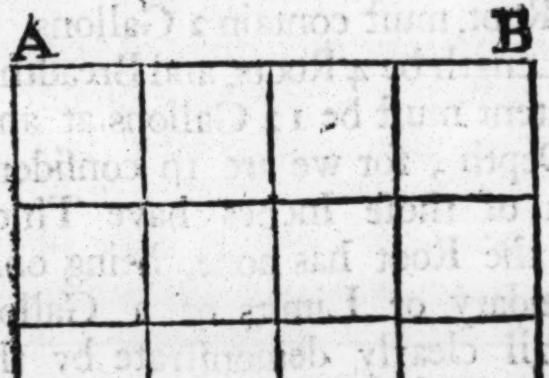
I Have often admired that thoſe Great Artiſts in Gauging, who have taken ſo much Pains in collecting of Lines of Numbers, Segments, and others, which were found out many Years before they were Born, and placed them upon ſliding Rules, which went by their own Names, could never think of any Thing new to facilitate the Practice of Gauging, and to leſſen the Labour of the Painful Officer, who I have often known ſpend an Hour in a

Malt, or Brew-houſe, when that Work might have been done in a Tenth Part of the Time, had they known any other Way than the old Practice of breaking their Work into ſmall Pieces, and putting it together again into greater, as if a Man ſhould compoſe an Integer of $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ and $\frac{1}{4}$, when he might as well have call'd it 2 halfs at once.

Though 282 is order'd to be the Standard Quantity of Inches in a Gallon, what Occaſion has a Man to take Dimenſions by Inches, and then to divide the Number of Superficial Inches, produced by Multiplication, by the Sum of the Inches in a Gallon, when he may do it quicker, and eaſier, by taking the Dimenſions by the Sq. Root of 282, and not by Inches, the Fractions of 282?

For ſince 282 Inches was fixt upon to be a Gallon, it conſequently follows that this Number of Inches muſt, or ought to be contain'd in ſome Square Veſſel, whoſe Side is the Sq. Root of 282, or elſe that it is to be ſuppoſed a Parellapipedon, whoſe Baſe is one Square Inch, and Length 282 Inches. So that any Veſſel, whoſe Length is twice the Root of 282, and Breadth once the Root, muſt contain 2 Gallons. Or if its Length be 4 Roots, and Breadth 3, its Content muſt be 12 Gallons at an Inch in Depth; for we are to conſider, that each of thoſe Inches have Thickneſs, but the Root has none, being only the Boundary or Limits of a Gallon, as I ſhall clearly demonſtrate by the following Scheme.

I find the Root of 282 to be 16,796, *&c.* ſo that I ſuppoſe the Side of each Diviſion of this Scheme to be the Length in Inches of this Root.



A B = 67,184.
A C = 67,184.

Suppoſe this to be a Square Cooler, 67,184 Inches each Way, which is equal to Four Roots; the *Area* found by

Squaring the Inches, will be equal to the *Area* found by Squaring the Number of Roots, *viz.* 16 Gallons, for the Square of 67,184, divided by 282, produces 16 Gallons, and ſo does Four Squared.

This Line of Square Roots is adapted to finding of *Areas* at an Inch deep, but will not ſerve for Solids.

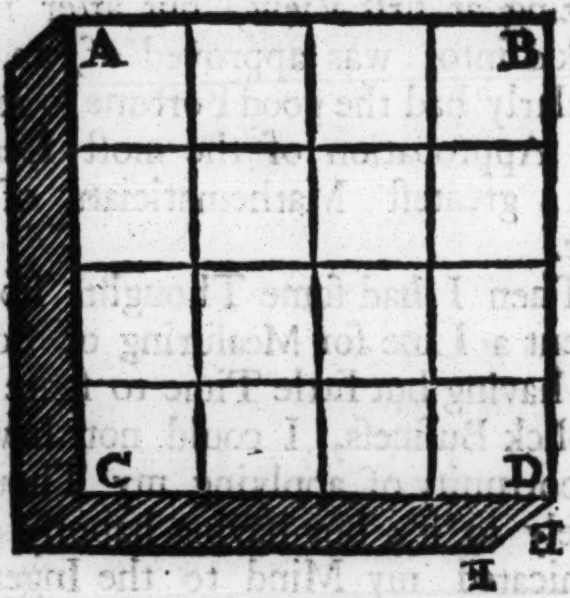
Therefore I make the *Cube* Root of 282, the Side of a *Cube*, which contains a Gallon, or the Side of ſuch a *Cube*, which is equal in Solid Capacity to a Gallon, *viz.* 6,56.

Note, That 282 is a ſurd Number, to which there can be found no exact Root, but that which is found will make no viſible Error in common Practice.

Since 6,56 Inches is the *Cube* Root of 282, and 16,796 is the Square Root, if any Ruler be divided into ſuch Diviſions as ſhall be of equal Lengths to either of thoſe Roots, and thoſe Lengths repeated as often as there is Occaſion, and Sub-divided into Decimal and Centeſimal Parts, ſurely the Dimenſions

of any Streight-lined Figure, bei
taken by that Ruler, as if they we
taken by Inches, and Multiply'd in
one another without any Diviſio
will produce the *Area*, or Solidit
as the Nature of the Thing requires
ſo that there will be no Occaſio
for long Multiplications, nor Diviſion.

Suppoſe the following Figure t
be a *Cube*, whoſe Sides are each equa
to Four Times the Length of th
Cube Root of 282, *viz.* 6,56, A, B
C, D, the Surface, and E, F, th
Depth of the *Cube*.



It's plain that Four Times 6,56, ſqua-ed, and multiply'd by Four times 6,56, he Depth, and divided by 282, will e 64 Gallons, equal to Four ſquared, ultiply'd into Four, or the *Cube* of our equal to 64.

About Three Years ago, at the latter nd of my Treatiſe of Meaſuring, I ad-ed a ſmall Taſte of the Line of Surfa-

ces, apply'd to Gauging without Inc
or Diviſion, which was ſomething ſ
prizing at firſt View, but after it w
look'd into, was approved of, and p
ticularly had the good Fortune to recei
the Approbation of the moſt Learn
and greateſt Mathematician of t
Age.

Then I had ſome Thoughts how
invent a Line for Meaſuring of Solid
but having but little Time to ſpare fro
Publick Buſineſs, I could not have
Opportunity of applying my Though
cloſely to it; but having happily com
municated my Mind to the Ingenio
and unknown Mathematician, Mr. *Jo*
Harwood, of *Windſor*, whoſe exceller
Knowledge in that Noble Science is grea
I then with Pleaſure went through th
Labour of finding Lines for Streigh
lined and Curved Solids, and have mad
uſeful Applications thereof by famili
Examples; for thoſe who have a Min
to make Uſe of them, and leaves ever
Man to his own free Will, to uſe whic
Way he pleaſes, having both Ways de

monſtrate

nſtrated as plain as poſſibly can be, only beg of the Reader to pardon Familiar Stile, the Errors of the eſs, and receive the Good Intentions Well Wiſhes of the Reader's

Moſt Humble Servant,

J. L.

The Rules are made by that Ingenious Artiſt, Mr. *William Haddon*, in *Giltſpur-ſtreet*, near *Newgate*, who I think is one of the beſt Performers in making Mathematical Inſtruments that we have in the Kingdom.

MEMORANDUM

That if the Honourable Commiſſione of the Exciſe will be pleaſed to appoint their General Gauger to prov this new-invented Way of Gauging I am ready to attend him, and ſhall then prove by the Practice that I, or any Gauger, may Gauge Twenty Backs this Way, while another ſhall Gauge One the other Way, and ſhall come exactly to the ſame Quantity of Liquor, either in Area *or* Content, *conſequently of Uſe to the Management.*

I had the Honour to ſerve in the Exciſe ſome Years, in which Time I invented the Line of Areas *by Roots, but never arrived to the Lines of* Solids *till now.*

THE New ART OF Gauging and Meaſuring WITHOUT nches or Diviſion.

I Think it is a General Topick, that whatever Rule is leaſt intricate, and ſooneſt perform'd, is preferable that which is more troubleſome and tedious, provided it anſwers the ſa End, and correct.

That Noble Invention of Logarith has been of ſuch excellent Uſe for Conciſeneſs in Arithmetical Calculatio that the Name of the Author, or fi Inventer, is Recorded amongſt the fi Rank of the moſt Learned of Ages, and his Logarithms are uſed in Nations.

Before the Logarithms were invente there were ſuch a Number of troubl ſome Operations required, that it ma the Work infinitely more tedious th it now is, but as ſoon as ever Logarithn appeared in the World, they were re dily imbraced by the Learned, and diſperſed into all Nations, that they b came the Darling of Perſons Mathem tically inclined, and the old tedio Ways quite left off.

I have no other Reaſon to doubt m Inventions being received, but that has the Misfortune of being Calculat for the Uſe of thoſe of a meaner Ge us, who will be very unwilling to Lea

o

off their old Way, tho' Ten times more tedious, and abundantly more ſubject to Errours than this.

Surely thoſe who underſtand the Rule of Practice in Meaſuring, by taking the Dimenſions in Feet and Inches, and multiplying Croſs-ways, as in the following Example, inſtead of taking the Dimenſions in Inches, will believe the ſhort Way the beſt.

Examples.

	Inches.	Feet.	Inch.
Length———	54	4	6
Breadth ———	30	2	6
	144)1620(11.25	9	0
	180	2	3
	36.0		
	720	11	3
	000		

But if there can be found a ſhorter Way than this it will be ſtill better, and more to be valued by the dexterous Artiſt.

My Deſign is to introduce a Way of Meaſuring Surfaces and Solids without Inches, Feet or Diviſion, only by Lines compoſed of the Sides or Boundaries of a *Cubical* Gallon, being equal to the *Square* or *Cube* Roots inſtead of Inches; by which Lines any Perſon may Meaſure or Gauge any Surface or Solid with Ten times the Eaſe, and in the Twentieth Part of the Time, they can do it by the Line of Inches, as will plainly appear by working the ſame Queſtions both Ways, and comparing them.

I ſhall firſt begin with ſome Examples in Gauging, by which it will appear what Time is ſpent in the Way now uſed, and how eaſie, and ſpeedily, it may be perform'd this Way.

The Deſcription of the Line for taking Dimenſions of Squares, or Oblongs, &c. *and finding the* Areas *at an Inch deep in Beer or Wine Gallons.*

This Line is only fitted for Beer *Areas.*

THE Law hath directed that a Gallon of Beer ſhall contain 282 *Cubical* Inches, and a Gallon of Wine 231, that is, ſuch a Square, Oblong, or other Form of Veſſel, whoſe Superficial *Area* of the Baſe is 282 Inches, or 231, and Depth One Inch, or a Parallelepipedon of One Inch Square, and 282 Long, contains juſt a Gallon.

Then ſince 282 is the Superficial *Area* of the Baſe of a Square, which holds a Gallon of Beer at an Inch in Depth, that Square muſt have a Side equal to the Square Root of 282, *viz.* 16,796, ſo that any Number of thoſe

Roots being multiply'd into one an ther, will produce the ſame Number Gallons, without any Diviſion, as Inches were multiply'd into Inches, an divided by 282.

Now I divide the Rules into Squa Roots thus.

I ſet off upon a blank Ruler th Length of 16,796, and that Length i call'd Number 1, being the Firſt Root the Second Length of the Root will b 33,6; the Third 50,4; and the Fourt will be 67,18, *&c.* and every one o thoſe Roots, or Lengths, are divide and ſub-divided into Tenths, and Hun dred Parts.

Examples of Squares, Oblongs, and Triangles.

A Triangle is only half a Square o Oblong, and is meaſured as ſuch, b finding a true Perpendicular, and multiplying half the Perpendicular into th whole Baſe, and dividing as if it were Square or Oblong.

Suppoſ

Suppoſe the Dimenſions of an Oblong ooler were taken in Inches, and the ngeſt Side be 50,4, the ſhorteſt Side 3,6. Multiply 50,4 by 33,6, and ivide by 282, and the *Area* will be Six allons.

Example.

By Inches.	By Roots.
ngth——50,4	Length=3
eadth — 33,6	Breadth=2
3024	=6 Gal.
1512	
1512	

282)1693,44(6,00 Gallons.
144

But the Dimenſions being taken by e Line of Roots, the longeſt Side will Three, equal in Length to 50,4 on e Line of Inches, and the ſhorteſt Side ll be Two, equal to 33,6 Inches, en theſe Dimenſions, Two and

Three being multiply'd as above, giv
Six the *Area*, equal to the other, whic
was taken by Inches, and requires bu
one Tenth Part of the Figures, an
without any Diviſion.

We ſhall ſuppoſe another Back to b
564,8 in Length, by the Line of Inc
es, which is equal to the Numb
33,62 on the Line of Roots, and i
Breadth 336 Inches, equal to the Nu
ber 20,01 on the Line of Roots.

Examples in Inches.

Length——564,8
Breadth—— 336

33888
16944
16944

282)189772,8(672,9 *Area* in Gal
2057
832
2688
150

Example in Roots.

Length——33,62
Breadth — 20,01
———
3362
6724
———
672,7362 *Area* in Gallons.

This is one of the very largeſt Backs in *England*, and yet the Difference is not Two Tenths of a Gallon, which may be occaſion'd by the Difference of taking the Dimenſions, for the Rule is grounded upon an undeniable Theorem.

Multiply thoſe *Areas* by the Depth in Inches, and Tenths of an Inch, and the Product is always the Content in Gallons and Parts.

I doubt not but that the unprejudiced Artiſt will grant that this Way is not only the quickeſt, but the moſt correct, by Reaſon it is not ſo liable to Errors, as that which requires a long Multiplication and Diviſion. And it is ſure, that as Square or *Cube* Roots are only the Boundaries of Quantities, Quantities ought to be meaſured by Roots, and not by Inches, which are only Fractions of Roots.

The Deſcription of the Line for taking the Dimenſions of Circular Veſſels, and finding the Areas.

AS 282 is allow'd to be the Diviſor for Squares or ſtreight Lined Figures in Beer Meaſure, and 231 for Wine, ſo is 359,1025 the Diviſor for Circular or Curved Veſſels, and 294 for Wine.

359,1025 is the Square of the Diameter of that Circle which contains 282 Inches in Surface, equal to one Gallon at an Inch in Depth.

And the Square Root of 359,1025 is 18,95, of which Length I make my firſt Diviſion for Beer Meaſure on the Rule, for my Line for taking the Dimenſions of Circles, and call it 1, and the Second Diviſion or Length, I make at 37,9, my Third at 56,85, *&c.*

Example of Cylindrical or Oval Veſſels.

An Oval muſt be reduced to a Cylinder, by multiplying the ſhorteſt Diameter into the longeſt, and the Square Root of the Product is the Mean Diameter, then work as if it were a Cylinder.

Suppoſe the Diameter of a Cylinder to be 37,9, the Dimenſions being taken by the Line of Inches.

Example

Example by the Line of Inches.

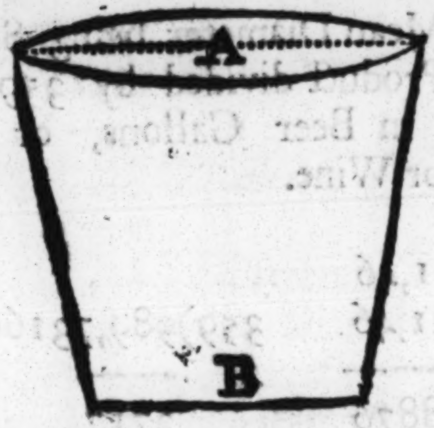
37,9 } Square the Diam. divide
37,9 } by 359 for Beer, and 294
—— for Wine, or multiply ¼
3411 the Circumf. by ½ the
2653 Diam. and divide by 282,
1137 or 231, which will an-
——— ſwer the ſame.
359)1436,41(4,00 *Area* by Gallons.
0041

Example by the Line of Roots.

The Diameter being taken by the Line of Square Roots, it will be Two Roots, equal to 3,79 Inches.

Diam. 2
2
—
4 *Area* in Gal. equal to the other.

To find the *Area* of the Mean Diameter of the *Fruſtum* of a *Cone*.



The Mean Diameter between A and B near enough, in common Practice is found thus.

Add ⅓ the Difference of the Two Diameters A and B to the leſſer Diameter, and that is the Mean Diameter.

Example in Inches.

A=36,4
B=29,0

7,4 Difference of Diameters.

2,46 ⅓ of the Difference.
29

31,46 Mean Diameter.

This Mean Diameter being Squared, and the Product divided by 359, gives the *Area* in Beer Gallons, or divide by 294 for Wine.

```
   31,46                          Area.
   31,46        359)989,7316(2,756
   ---------        -------
   18876            2717
  12584             -------
  3146               2043
 9438                 2481
 ---------             327
 989,7316
```

Examples in Roots.

To find the Mean Diameter by the Line of Square Roots, work as in Inches, thus.

A 36,4 Inches on the Line of Inches, is equal in Length to 1,92 on the Line of Roots; B 29 Inches is equal to 1,53 on the Line of Roots; then,

A

A=1,92
B=1,53

,39 Difference of Diameters.

,13 $\frac{1}{3}$ Difference.
1,53 Leſſer Diameter add.

1,66 the Mean Diameter.
1,66

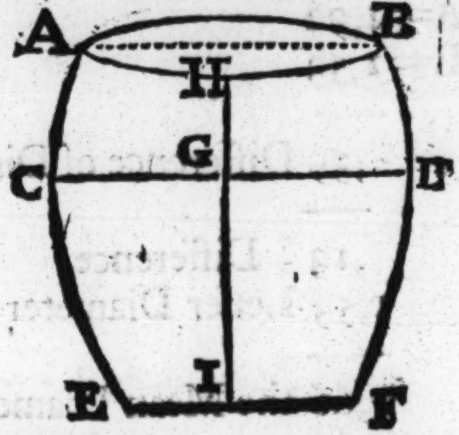
996
996
166

2,7556 the *Area* in Gallons.

To find the Area *of the* Fruſtrum *of a* Spheroid.

Firſt find the Mean Diameter thus, Subſtract A B from C D, and multiply the Differ. by ,7, add the Prod. to the leſſer Diam. and that's the Mean ; do the like with C D and E F.

Find



Find a Mean between A B and C D, and another between C D and E F, as directed above.

A B=26	C D=30	C D=30
C D=30	A B=26	E F=24
E F=24	———	———
G H=20	4	6
G I=25	,7	,7
	———	———
	2,8	4,2
	26	24
	———	———
	M. 2,88 Mean.	N. 28,2 M.

M the Mean of the upper *Fruſtum*, and N the Mean of the lower one.

The Square of each Mean Diameter divided by 359, produces the *Fruſtum* in Beer, or 294 for Wine Gallons.

The half of the Sum of thoſe Two Means may paſs in Practice, which will be 28,5; but the true Geometrical Means is the Square Roots of the Rectangle of the Diameters, A B into C D, and C D into E F; but the Difference of the Diameters from Bung to Heads, are ſo many and various in ſuch Veſſels, that it is very Erroneous to reduce any Curved Cask, other than ſuch Casks as are near a Cylinder to a Mean Diameter, becauſe the Difference of all their Dimenſions are ſuch, that ſuppoſe Three Inches from the Head to the Liquor's Surface were wet, and it ſtood with its *Axis* Perpendicular to the Horizon upon that End, there would be Five Gallons in it; but ſuppoſe that it were reverſed, and ſtood upon a Head equal to the Diameter at the Bung, and Three Inches wet, then would there be near Eight Gallons in the Cask or Tub; which I think is ſufficient to prove there can be no true Mean in Practice, for a thorough *Area*; but taking 4, 5, or 6, ſeveral Diameters, and from them ſo many *Areas*, is better.

We ſhall not trouble ourſelves about *Areas* in this Caſe, but ſhall find the Solid Capacity by Inch Meaſure, and then by Root Meaſure of the Middle *Fruſtum* of a *Speroid*, ſuch as a Hogſhead. For having agreed upon your Mean Diameter, work as in the foregoing Rule, by Squaring, Dividing, or Multiplying, either by Inches or Roots.

Thoſe Examples are ſufficient to ſhew you the Uſefulneſs of the Line of Roots, and the Uſeleſneſs of Inches in taking Dimenſions.

I ſhall now begin to ſhow you the Uſe of my new-invented Line of Solids, never thought of by any Perſon before myſelf, and ſhall apply it to Gauging all manner of ſtreight-lined Figures, ſuch as Triangles, Squares, Oblongs, Polygons, Rhombuſſes, Trapezias, and Priſms.

The

The Deſcription of the Line of Solids, and its Uſe in Gauging Streight-lined Figures.

AS the Diviſions on the Line for taking the Dimenſions of Streight-lined Veſſels, to find their *Areas*, are the Square Roots of 282, ſo this Line is compoſed of *Cube* Roots of 282, or, 231.

For if Square Superficial Meaſuring is to be perform'd by Square Boundaries, ſo Solid Meaſuring muſt require *Cubick* Boundaries.

Therefore I have made the firſt Diviſion on the Line equal to the *Cube* Root of 282, *viz.*

6,56 and call it 1
13,12 ====== 2
19,7 ====== 3
26,3 ====== 4, &c.

and divided it into Decimal and Centeſimal Parts, as before, ſo that I uſe it as if it were divided into Inches and Tenths, both in taking of Lengths, Breadths, and Thickneſs.

As thus, ſuppoſe I were to know how many Gallons of Beer a Square Cooler, or Tun of 52,48 Inches deep, and each Side 65,6 Inches long would contain, the Dimenſions being taken by the Line of Inches.

I Multiply the Length of the Side by its ſelf, and multiply the Product by the Depth, and that Product I divide by 282, or 231, and the Quotient is the Content.

```
            65,6
            65,6
           ------
           3936            Cont. in a Gal.
          3280     282)225840,3328(800
         3936              1844
        --------            1620
        4303,36              2103
           52,48              1293
      -----------              1652
        3442688                 2428
       1721344                   172
      860672
     2151680
    -------------
     225840,3328
```

I ſhall work this Example as if the Dimenſions were taken with the Line of Solids, and compare them together, to ſee the Difference as to the Number of Figures, and Trouble in the Operation, which I hope will convince any Artiſt of the Uſefulneſs and Excellency of thoſe new-invented Lines.

65,6 Inches being the Length of the Side, is equal in Length to the Number 10 on the Line of Solids; and the Depth 52,48 Inches, is equal to the Number Eight on the Line of Solids.

Now ſuppoſe I have taken my Dimenſions by the Line of Solids, I ſet it down thus,

Side = 10 on the Line of Roots.
10

100 the Square.
8 the Depth.

800 Con. in G. equal to the other.

So that you ſee here are but Eleven Figures, and there are in the other Way 102, beſides the Trouble of Two long Multiplications, and One long Diviſion.

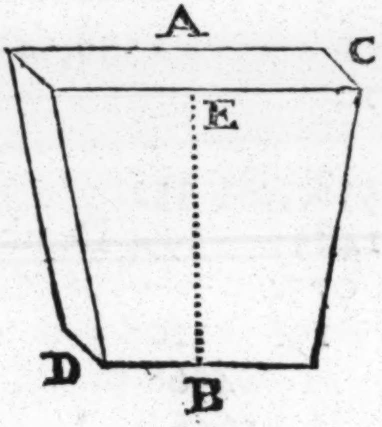
To find the Solid Content of the Fruſtum *of a Square Pyramid.*

A=C=45,8 each, } by the Line of Inch
B=D=32,8 each, }

Subſtrac

Subſtract B from A, add $\frac{1}{3}$ of the Square of the Difference to the Reſtangle of A and B, and you have the *Area* in Inches, divide by 282 Inches, and you have the *Area* in Gallons, multiply by the Depth for the Content.

A the Side at the Top, B the Side at the Bottom, E B the Depth.



```
A=45,8              45,8
B=32,8              32,8
  ----              ----
  13,0              3664
  13,0              916
  ----            1374
  39              --------
 13               1502,24
  ----              56,3
3)169(56,3        --------
   19             1558,54 theSuperf.
    10

282)1558,54 ( 552
     1485     39,4 Depth.
      0754    ----
       196    2208
             4968
            1656
            --------
Content 217,488 Gallons.
```

By the Line of Solids.

Longeſt Side A is 45,8, equal on the Line of *Cube* Roots, to 7, the ſhorteſt

Side B is 32,86, equal to 5 on the Line of Roots, and the Depth 39,4, is equal to Six on the Line of Solids or Roots.

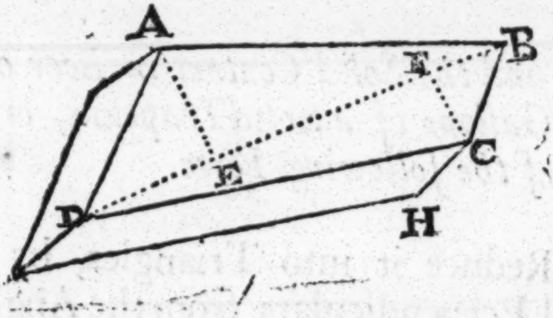
A=7	7
B=5	5
2	35
2	1,3 = $\frac{1}{3}$ the Differ.
3)4(1,3 = $\frac{1}{3}$	36,3
1,0	6
	217,8

To find the Solid Content in Beer or Wine Gallons of a Solid Trapezia, or a Back of the following Form.

Reduce it into Triangles, by letting fall Perpendiculars from the Angles upon the neareſt Point of the oppoſite Side, or common Baſe, and find the Superficial *Areas* of each Triangle, by multiplying the half Baſe by the whole Perpendicular, divide the Sum of them by 282, or 231, and multiply the Quotient by the Depth, and you have the Solid Content in Gallons.

Example by Inches.

A B C D a Trapezia, B D the Common Baſe, A E and F C Perpendicular, C H the Depth.



Half

```
Half AB 39,4 is 19,7   DB 118,2     DB 118,2
Half F G 26,3 is 13,1       19,7          13,1
       G H 26,3 =13,1      -----        ------
                            8274          1182
                           10638         3,546
                           1182          1182
                           -----        ------
                         2328,54       1548,42
                         1548,42
                         -------
                    282)3876,96 ( 13,74
                        1056      26,3
                         2109     -----
                         1356      4122
                          228     8244
                                 2748
                                 ------
           the Content in Gal.  361,362
```

A E	39,4 Inches	Equal to	6	on the Line of Solids.
F C	26,3 Inches		4	
G H	26,3 Inches		4	
B D	118,2 Inches		18,07	

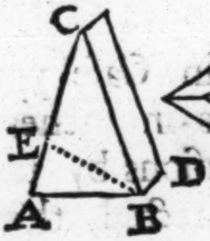
Half of *6* is 3
Half of 4 is 2

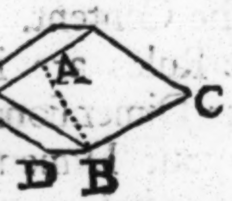
18,07	18,07
2	3
36,14	54,21
54,21	
90,53	
4	
361,4 Content.	

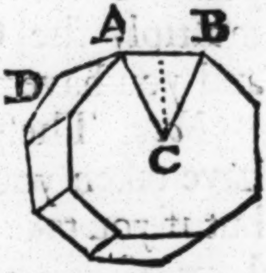
The Content both Ways are equal only in the Decimal Parts, there are near 05 Difference, all which are occaſion'd by Remainders.

All other Streight-lined Figures, ſuch as Poligons, Priſms, Rhombuſſes, *&c.* are meaſured by reducing them into Triangles, Oblongs, or Squares, by raiſing or letting fall Perpendiculars, as you ſee in the Trapezia, and the annexed Schemes.

Priſm.


Priſm.


Rhombus


Poligon.

From the Center of a Poligon to each Side, divide its Surface into Triangles, as A B C, multiply the Superficial *Area* of one Triangle by the Number of Sides, and divide by 282, or 231, for the *Area* in Gallons of Beer or Wine, multiply that by the Depth B D.

For a Rhombus multiply the Perpendicular A B into the Side C B, and you have the *Area* in Inches, divide by 282, you have the *Area* in Beer Gallons; or by 231 for Wine, multiply by the Depth B D, and you have the Content in Gallons.

For a *Priſm*, find the *Area* of the Triangular Surface A B C, by multiplying the half of the Perpendicular B E into the Side A B, and divide that Superficial *Area* by 282, or 231, and multiply the Quotient by the Depth, and you have the Content.

Note, Theſe Rules are in Caſe you have taken your Dimenſions in Inches; but if you take your Dimenſions by the Line of Solids, as before directed, then you only multiply the Superficial *Area* into the Depth, and you have the Solid Capacity in Gallons without any Diviſion.

As for Example, ſuppoſe the Dimenſions of the *Rhombus* be taken by the Line of Solids, and were found to be

A B=6 which is equal to 39,4 Inches.
C B=7 which is equal to 45,8 Inches.
——
42
B D=6 which is 39,4 Inches.
——
252 Gallons of Beer is the Content.

If the Dimenſions had been taken in Inches, it would have required Two Multiplications, and One Diviſion, and at leaſt 62 Figures, whereas this is perform'd by Two ſhort Multiplications, and in Eight Figures.

The Content of a Pyramid and *Cone* are found by multiplying the *Area* of the Baſe by $\frac{1}{3}$ of the Altitude, and dividing by the proper Diviſors for Beer, Wine, Stone or Timber, *viz.* 282 for Beer, 231 for Wine, or 1728 for Stone or Timber. *Note*, Thoſe are the proper Diviſors, when the Square *Areas* in Inches are found. But,

It's cuſtomary in a *Cone* to Square the Diameter of the Baſe, and multiply by $\frac{1}{3}$ of the Height, and divide by 359, or 294 for Beer or Wine, or multiply the Square of the Diameter by the whole Height, and divide by 1077 for Beer, or 882 for Wine.

To find the Content of the *Cone* by the Line of Solids, the Diameter of the Baſe being Seven Roots, and Height 12 Roots, or Numbers on the Line of Solids.

Subſtract the Square of ½ the Baſe, from the Square of the Slaint Height, and the Remainder is the Square of the Perpendicular.

I find the *Area* of the Baſe by multiplying half the Circumference by half the Diameter; thus multiply the Diameter by 3,14, and you have the Circumference. Therefore,

3,14

Diam. =7

21,98 the Circumference.

10,99 ½ the Circumference.

3,5 ½ the Diameter.

5495

3297

38,465 the *Area* of the Baſe.

12 the Depth.

76930

38465

461,580 the Con. in Beer Gal.

If this Operation had been perform'd by Inches, it would have required above Six times the Number of Figures; but this being a Circular Solid, the following Line, which is adapted to Circular Bodies only, would have perform'd it more eaſily.

The Deſcription and Uſe of the Circular Line of Solids, in Gauging Curv'd-lined Figures, ſuch as Casks, when the Dimenſions are found by Squaring the Diameters.

SINCE 359,1025 is the Square of that Diameter, which holds a Gallon at an Inch in Depth of Beer, equal to 282 Solid Inches, and conſequently the proper Diviſor for dividing the Squares of Diameters by, to find the *Area* in a Square Capacity; I have aſſerted that the Square Root of theſe Numbers, *viz.* 17,14, and 18,95, are proper to be the firſt Diviſions on the Line for finding *Areas*. Therefore the *Cube* Root of 359,1025, ought to be the firſt Diviſion on the Line of Solids for Curved-lined Veſſels for Wine or Beer Meaſure, and is found to be 7,1075 for Beer, the *Cube* Root of 359,1025, or 6,56 being the *Cube* Root of 294 for Wine, as near as need be in Practice, or can be brought from a Surd Number; and are the Sides of ſuch *Cubes* which will reconcile a Square without, to a Square equal: For the Squaring the Diameter gives a Surface too large, therefore the Diviſor 359 was uſed, being the Square of the Diameter of ſuch a Circle, which held a Gallon on an Inch, conſequently the *Cube* Root of that muſt be proper.

The

The Diviſions on the Circular Line of Solids, are theſe for Beer Meaſure.

THE firſt Diviſion is the Length of 7,1075 Inches, being the *Cube* Root of 359,1025 Markt			1	Roots.
The 2d Diviſion is	14,2156	Markt	2	
The 3d Diviſion	28,3225		3	
The 4th Diviſion	28,43		4	
The 5th Diviſion	35,5375		5	
The 6th Diviſion	42,645		6	
The 7th Diviſion	49,7525		7	
The 8th Diviſion	56,86		8	

The Firſt Diviſion for Wine Meaſure is 6,65, the *Cube* Root of 294.

Take the Dimenſions of any Cask, with the Lines divided ſo as if you were taking it in Inches, and multiply only as is hereafter made plain by Examples, and you have the Solid Content of any Cask as exact as if it were taken in Inches, and Ten Times more eaſie.

There are Three Varieties of Shapes or Forms of Casks, *viz.* the Middle *Fruſtum* of a Spheroid, the *Fruſtum* of a Parabolick Spindle, and the Middle *Fruſtum* of a *Hyperbola*; I deny any Cask to be the Middle

Fruſtum of Two *Cones*, abutting on a common Baſe, as ſome aſſert.

To Gauge the Middle Fruſtum *of a Spheroid, whoſe Head Diameter is* 21,32 *Inches, equal to Three on the Line of Circular Solids. Bung* 24,87 *Inches, equal to* 35 *on the Line of Circular Solids, and Length* 35,5 *Inches, equal to Five on the Line. The Content is requir'd by working by Roots and Inches, and comparing them.*

Head —— 21,32 equal on the New Line to 3
Bung C D 24,87 equal to ——— ——— 3,5
Length E F 35,5 equal to ——— ——— 5

TO Twice the Square of the Bung Diameter, add Once the Square of the Head, multiply that Sum by the Casks Length, and divide by 1077 for the Content in Beer, if you Work by Inches; but if by Roots, and have taken the Dimenſions by the Line of Circles, then the Double Square of the Bung, and the Square of the Head Diameter, added and multiply'd into the Length, gives the Content at once.

Example by Inches.

Bung Diam.	24,87	Head Diam.	21,32
	24,87		21,32
	17409		4264
	19896		6396
	9948		2132
	4974		4264
	618,5169		454,5424
	2		

1237,0338 Doub. Sq. of the Bung Diam.
454,5424 Square of the Head Diam.

1591,5762
35,5 Casks Length.

79578810
79578810
47747286

1077)56500,955(52,461 Beer Gallons.
2650
4969
6615
1535
458

Example by the Line of Circles.

Bung Diam. 3,5
Head Diam. 3
——
10,5
Length —— 5
——
52,5

This Way requires but 10 Figures, and the other 147 at leaſt.

There are other Ways of Gauging Casks, ſuch as Mr. *Oughtred*'s and Mr. *Caſwell*'s, but take which Way you pleaſe, the Dimenſions being taken by the Line of Solids, and thoſe Dimenſions being multiply'd into one another, as before, infallibly reſolves the Queſtion.

To find the Ullage of a Cask Lying by Inches, and by the Circular Line of Roots. Suppoſe the Middle *Fruſtum* of a Spheroid, whoſe Content is 57 Gallons, Bung Diameter 25,8 Inches, Wet Inches 17,2, the remaining Liquor required.

Add a competent Number of Cyphers Decimal Ways to the Wet Inches, and divide by the Bung Diameter, and if the Quotient exceed 50, add to it a Third Part of what it is above 50, but if under 50, ſubſtract a Third Part of what it wants of 50, multiply the Sum by the Content, and you have the Remaining Liquor.

Example

Example by Inches.

25,8)17,200) ,66
1720 ,50
172 ——
3),160),053
Content — 57 Gallons 10 ,66
Bung Diam. 25,8 Inch. 1 ——
Wet Inches 17,2 ,713
57
——
4991
3565
——
40,641 Gallons.

Example by the Circular Line of Solids.

Work the ſame Way when you take your Dimenſions by the Line of Circular Solids, as if you had taken them by Inches.

The Bung Diameter being taken by the Circular Line, will be = 3,63, and Wet Inches will be 2,4.

3,63)2,4000 (,66
2220 ,50
42 ——
3),160 (,053
10 ,66
——
,713
57
——
4991
3565
——
40,641

Note, That the *Cube* Root of one Number being divided by the *Cube* Root of another Number, brings forth the ſame Quotient as if the Numbers were divided, and that Quotient Cubed.

To find the Vllage, or remaining Liquor of a Cask ſtanding.

There are ſeveral Ways preſcrib'd for this Ullage, but I dare ſay none juſt, therefore I would adviſe the honeſt Gauger to conceive it as Two Quantities, One Quantity being from the Bung to the Head whereon it ſtands, and the other Quantity from the Middle of the Bong to the Liquors Surface, computing that below the Middle of the Bung to be half of the Content, and Meaſure the other as the *Fruſtum* of a Spheroid, and add the Two Quantities together for the remaining Liquor, or Meaſure the *Vacuum* as the *Fruſtum* of a Spheroid, and ſubſtract it from the whole Content.

I have wrought all the foregoing Propoſitions as if they were Beer; and becauſe Wine is ſeldom or never put into any other Veſſels than thoſe of Circular Form, ſuch as Hogſheads or Pipes, *&c.* I have only made a Line for Circular Solids, beginning at 6,65, being the *Cube* Root of 294, *viz.* the Square of ſuch a Diameter, which holds a Gallon of Wine at an Inch Deep.

The Diviſions on the Circular Line of Solids for Wine Meaſure.

	Markt	Roots
THE Firſt Diviſion is in Length — —	6,65	1
The Second ———	13,3	2
The Third ———	19,95	3
The Fourth ———	26,6	4
The Fifth —— ———	33,25	5
The Sixth – ———	39,9	6
The Seventh. ———	46,55	7

Suppoſe a Wine Cask, whoſe Bong-Diameter is 26,6, and Head 19,95, and Length 39,9, what's the Content. This would require 67 Figures wrought by Inches.

By the Circular Line of Roots.

Length 39,9 = 6
Bong 26,6 = 4
Head 19,95 = 3

Bong	4	3,7	Mean.
Head	3	3,7	
	—	25,9	
Diff.	1	111	
	,7	———	
	—	13,69	
Mean	3,7	6	Length.
		———	
		82,14	Gallons.

The

The Deſcription and Uſe of the Lines of Areas, *and Contents, for Gauging Malt, in Cyſtern or Floor.*

THE Square Inches allow'd to be in a Buſhel of Malt are 2150. The Square Root whereof is 46,36, of which Length is the Diviſions on the Line for taking *Areas* in Buſhels; and 12,9 is the Cube Root, and Length of each Diviſion on the Line of *Solids*, for taking Contents in Buſhels.

The Line of *Areas*. The ſolid Line of *Solids*.

Inches.		N. *of Roots.*	*Inches.*		N. *of Roots.*
46,36	=	1	12,9	Equal to	1
92,72	=	2	25,8		2
139,08	=	3	38,7		3
185,44	=	4	51,6		4
231,8	=	5	64,5		5
278,16	=	6	77,4		6
324,52	=	7	90,3		7
370,88	=	8	103,2		8
417,24	=	9	116,1		9
463,6	=	10	129,0		10

Theſe Lines may be put upon a Tape, for the Four-foot Rule will contain but One of the Lines for *Areas*, and not quite Two of the *Solids*.

Suppoſe I were to take the Dimenſions of a Floor of Malt with the Line of *Areas*, whoſe Length is 370,9, which is equal to 8 on the Line of *Areas*, By

By Inches.	By ſolid *Areas.*
370,8 Length.	8 = Length.
185,4 Breadth.	4 = Breadth.
14832	32 *Area.*
18540	
29664	
3708	
2150)68746,32(31,97 *Area.*	
4246	
20963	
15132	
082	

Now ſuppoſe a Steep of Malt, whoſe Sides are each 64,5 equal to 5 on the Line of *Solids*, and Depth 103,2 equal to 8 on the Line of *Solids*.

By Inches.	By the Line of *Solids*.
64,5	5
64,5	5
3225	25
2580	8
3870	
	200
2150) 4160,25 (1,93	103,2
20102	1,93
7525	
1075	3096
	9288
	1032
	199,176

This Difference is occaſion'd by the larg
Remainder, 1075 in the Diviſion.

Board and Timber Meaſure, by Pen an Gunters Line, and by the new Line o Surfaces and Solids.

THE Lines of Surfaces and Solids being al ready on every Rule which has a Line o Inches, I have no Occaſion to Treat of it other wiſe than to adviſe thoſe who have a Mind t have it ſo, that a Foot of 12 Inches ſhould b divided into 10 Parts inſtead of 12, and eac of thoſe ſubdivided into 10 Centeſimals; th your Work would be more eaſie than it no is: For the Parts of an Inch creates ſome Tro ble in the Working, whereas the Decimal D viſions includes theſe Parts, and they a wrought as whole Numbers, and the Decim are eaſily valued.

Example, A Square Piece of Timber 3 F 6 Inches each Way, equal to 3,5 Decimally the Rule of Roots; for I make 12 the C Root of 1728, the Firſt Diviſion on the Li and ſo repeat it as often as Occaſion ſerves.

Feet Inches.	Foot Tenths.
3,6	3,5
3,6	3,5
10,6	175
1,9	105
12,3	12,25
3,6	3,5
36,9	6125
6,1 $\frac{1}{2}$	3675
42,10 $\frac{1}{2}$ Content.	42,875 Content equal.

12 Inches is the Square Root of 144, the Superficial Inches in a Foot; and it is likewiſe the Cube Root of 1728, the ſolid Inches in a Foot; therefore being call'd 1, 2, 3, 4, *&c.* on the Rule, inſtead of 48 Inches, and each of thoſe Feet divided into 1, 2, 3, 4, 5, 6, 7, 8, 9, ſmaller Figures, it will be the moſt proper Inſtrument to take Dimenſions, either in Surfaces or Solids.

Of Surfaces. To meaſure a Square whoſe Sides are each 70 *Inches, or* 5 *Foot* 10 *Inches.*

Multiply one Side into the other, and divide by 144.

Vulgarly.

70
70

144) 5900 (34
580
4

Practically.	
Feet.	**Inch.**
5,	10
5,	10
29,	2
2,	11 = $\frac{1}{2}$
1,	11 = $\frac{1}{3}$
34,	0

The Practical Way is wrought thus, multiply 5 into 10 Croſs-ways, which makes 4 Foot 2 Inches, and then 5 Times 5 Feet makes 25, and 4 is 29 Feet; then, becauſe 10 Inches is the $\frac{1}{2}$ and $\frac{1}{3}$, take the $\frac{1}{2}$, and the $\frac{1}{3}$ of the firſt Line, and ſet down, then make a Sum, which is here 34 Feet.

By *Gunter*, As 1 is to 5$\frac{3}{4}$, ſo is 5$\frac{1}{4}$ to 34 Feet.

To meaſure a *Traperia*, take the Rule directed in Page 40 for the Surface, by dividing it into Triangles, and the Rule for Triangles is to let fall a Perpendicular from an Angle to the neareſt Place on the oppoſite Side, and $\frac{1}{2}$ the Perpendicular into the whole Side, and divide by 144.

Vulgarly.	Practically.	Decimally.
The Baſe. 42	Feet. Inch.	3,50
$\frac{1}{2}$ Perp. 33	3 : 6	2,75
——	2 : 9$\frac{1}{2}$ Perpen.	——
126	——	1750
126	7 : 0 = $\frac{1}{2}$	2450
——	1 : 9	700
144) 1386 (9,6	10$\frac{1}{2}$ = $\frac{1}{4}$	——
90.0	9 : 7$\frac{1}{2}$	9,625
36		

By *Gunter*, as 1 is 3 $\frac{1}{2}$, ſo is 2 $\frac{3}{4}$ to 9 Foot 7 $\frac{1}{2}$ Inches.

A Rhombus and Rhomboides are only Paralellagrams out of Square, therefore you are to let fall a Perpendicular from any one of the blunt Angles, to the neareſt Place of the oppoſite Side, and multiply the Perpendicular into the Side, and divide by 144, or work by Practice or Decimals as before, or by *Gunter* thus.

As 1 is to the Side, ſo is the Perpendicular to the Content.

To find the Content of a Circle.

Multiply half the Circumference by half the Diameter, and divide by 144, or by the Practical or Decimal Ways.

Multiply the Diameter by 3,14, and you have the Circumference of a Circle.

By *Gunter*, as 1 is to $\frac{1}{2}$ the Circumference, ſo is $\frac{1}{2}$ the Diameter to the Content.

In-

Inſtructions for Flat-work, as Painting.

As Rooms are of ſo many various Forms, I ſhall only give a General Rule in all Caſes.

Take a String, and apply one End of it to any one Corner of the Room, then meaſure the Room, going in to every Corner of the Molding with the String, until you come at the Place you began at; then Meaſure how many Feet and Inches the String is, and ſet down; then Meaſure the Height by the ſame Method, going into every *Angle* of the Molding, and ſet that down under the other; multiply the one by the other, and divide by 9, and the *Quotient* will be the Yards of Painting, or Wainſcot. If there be a Chimney, Meaſure the Chimney by the foregoing Rules, according to the Figure of it, and deduct from the whole.

Glaziers are to take the Depth and Breadth of all Windows that are alike in Inches, and make a Sum of all the Depths, and likewiſe of all the Breadths, and multiply one by the other, and divide by 144, and the *Quotient* is Feet; or by the Rule, as one is to the Heighth or Depth of the Whole, ſo is the Breadth of the Whole to the Content.

Meaſuring of Solids, *apply'd to* Timber *and* Stone

By *Gunter*, Set one Foot of the Compaſſes in 1, and extend the other to the Length of the Side in Feet and Parts, *viz.* 3 Foot and $\frac{1}{4}$; then turn your Compaſſes twice at that Extent, and the Point will cut 34 on the Rule, and ſomewhat more.

F I N I S.

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